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Arbitrage and the Price of Oil

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Arbitrage and the Price of Oil

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Abstract

The model simulated in this paper shows that falling interest rates contribute to rising oil prices. This occurs because oil producers treat oil in the ground as an asset and attempt to arbitrage differences between its rate of return and the interest rate. When calibrated to match observed data over the last two decades, model results indicate that this arbitrage behavior may have made the largest contribution to the pre-crisis boom in oil prices. Productivity driven growth shocks raise the oil price by about 70 percent, but this rises to 150 percent when falling interest rates are included.

JEL Classification: E37, F47, Q43

Keywords: Oil price, arbitrage, two region

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1. Introduction

The standard explanation for oil price rises which began in 2004 and culminated in 2008 is increased world demand combined with stagnating supply (International Energy Agency, 2007). This explanation focuses on the rise in demand, usually associated with strong economic growth in India and China. Figure 1 shows the time path of prices, of note is the steep rise beginning around 2004.[2] Given the magnitude and speed of this rise, it is unlikely that increased demand was the only factor (Hamilton, 2009). In particular, the macroeconomic reasons for stagnant supply may have contributed as well.

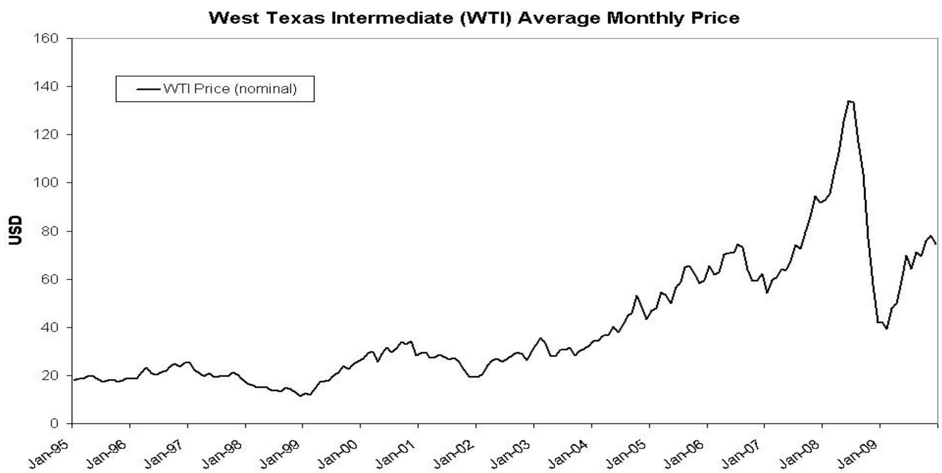

Figure 1

This paper looks at the macroeconomic factors behind the non-increasing supply during the rapid oil price rise. Specifically, it models a producer's supply decision in the face of changing interest rates. Research suggests that a relationship between oil prices and interest rates does exist and is also important. Mabro (1998) and Barsky and Kilian (2004) have argued that over the medium-run, interest rates will impact producer extraction and investment decisions. Both Akram (2008) and Frankel (2006) find evidence of a negative relationship between interest rates and the level of oil prices.

Simulation results from the model used in this paper show that a fall in interest rates can lead to

[2]Data are from United States Energy Information Administration (2010b)

an increase in the price of oil. This occurs because producer extraction decisions are sensitive to the level of interest rates, so any change in these rates will feed back to prices through the supply-side. Counterfactual simulations also assign a larger role in the price rise for falling interest rates than to increased demand. This goes against the conventional wisdom, and may indicate that low interest rates contributed to the price rise.

Theoretically, changes in interest rates will alter oil prices through producer extraction decisions if oil in the ground has value (Hotelling, 1931). Oil in the ground can have value because of scarcity, either because the stock of oil is finite, or because production capacity is fixed in the short-run. This may differ from the value it has as a final good or as an input to production above ground. From a producer's perspective, this additional value adds a facet to their extraction decision. Both the revenues from extracting and selling a barrel of oil, and the rate of return on oil in the ground need to be considered.

The model incorporates producer responses to changes in interest rates by constraining production capacity in the short-run. Oil in the ground can then be thought of as an asset in the producer's portfolio. The producer's problem is one of choosing extraction so that the rate of return on oil in the ground matches that of risk-free bonds in their portfolio. The model has two regions, four sectors, and incomplete asset markets. The rate of return on bonds is taken as a proxy for the interest rate.

The model is relatively large in order to better match the simulations with data. Using four sectors also allows consideration of multiple intermediate inputs, where substitution is possible between the inputs due to price changes. A larger model also expands the types of simulations which can be conducted. The model abstracts from uncertainty. Given that short-run oil production capacity is fixed, and the bonds used in the model are risk-free, abstracting from demand-side shocks should not drastically change the results.

Simulations from the model show that a fall in rates of return on bonds can lead to an increase in the price of oil. The effect of bond rates of return declining at their historical average on the oil price is greater than the effect of increased demand. The price increase is roughly 70% when only real GDP growth is taken into consideration, and over 150% when the declining rates of return are added. This result complements other explanations often cited for rising oil prices (see e.g. Hamilton

(2009) and Kilian et al. (2009)).

2. Common Explanations for the Oil Price Rise

The standard explanation is increased demand combined with stagnating supply (International Energy Agency, 2007). It holds that significant world economic growth, particularly in India and China, drove up global demand for petroleum products, while at the same time supply did not keep pace. Figure 2 shows world demand and percent changes in OPEC and Saudi Arabian production since 2000.[3] After about 2003, OPEC production was growing continually less as demand was rising. Strikingly, OPEC production fell between 2005 and 2007 in the face of rising demand.

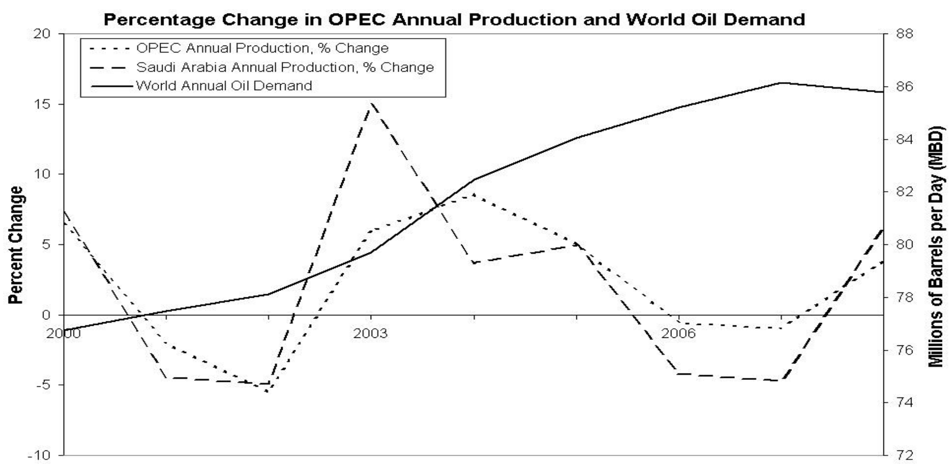

Figure 2

Another explanation for the price rises is a lack of OPEC spare capacity. This surplus production capacity, which is largest in Saudi Arabia, has historically helped to offset demand or supply shocks (Hamilton, 2009). Figure 3 shows OPEC and Saudi Arabian spare capacity since 1985.[4]

OPEC spare capacity has eroded since 2002 as demand has increased, leaving Saudi Arabia as the only country with significant surplus (United States Commodity Futures Trading Commission,

[3]Data used in the chart are from the United States Energy Information Administration (2010a).

[4]Data are from United States Energy Information Administration (2010a)

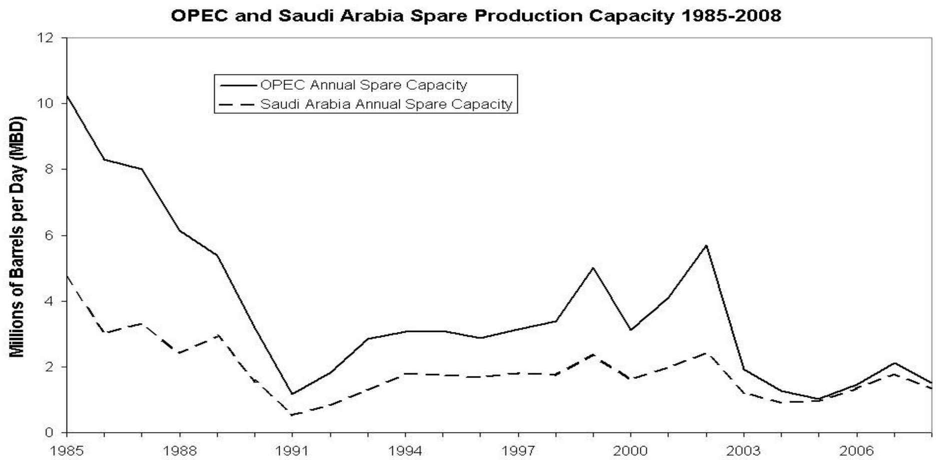

Figure 3

2008). Beginning in 2004, Saudi Arabian spare capacity actually rises. Figure 1 shows this is around the time nominal oil prices begin to rise steeply.

A third explanation focusing on arbitrage is examined in this study. Oil producers (particularly Saudi Arabia) may have withheld production by retaining oil in the ground because of low rates of return on relatively safe investments. This is consistent with the Hotelling theory, and also ascribes a role for global macroeconomic conditions in affecting oil supply as emphasized in Barsky and Kilian (2004). This explanation is formalized in the model put forth in the next section.

3. The Model

The model is intertemporal and has two regions that produce and trade in four goods, with a representative household in each region. The four sectors are: agriculture, manufacturing, services, and oil production. There are four factors of production: labor, land, and two types of capital (one for manufacturing and one for the oil sector). Labor is region-specific, but may move among all sectors. The other three factors are also region-specific, but stay within each industry. Intermediate goods are also used in production.

The representative household in each region owns the firms in each sector, and receives their capital rewards. The goods markets and all factor markets are perfectly competitive, allowing firms to take prices as given. As factor inputs, agriculture uses land and labor; manufacturing uses labor and capital; services uses only labor; and oil uses labor and capital. Returns on labor are assumed to equalize across industries. The discussion below focuses on general aspects of the model, with specifics on the asset portfolios and investment. The other portions are standard and relevant details can be found in Appendix I.

3.1. Macro Structure

The structure is intended to capture salient features of oil-producing and oil-consuming economies. Roughly, this is modeled as Middle East OPEC and the rest of the world (ROW). The OPEC region (Region 1), is an oil-producing economy with only oil as a significant export. It imports mainly manufactured goods, but some agricultural goods as well. The OPEC region is very small compared to the non-OPEC region (Region 2), which is a large service-oriented economy. The non-OPEC region imports significant amounts of oil, and exports manufacturing and agricultural products. Although its services sector is large, exports of services to the non-OPEC region are insignificant.

Each economy has a fixed savings rate in each period. Investors can be either forward-looking through perfect-foresight one period ahead, or backward-looking via adaptive expectations. In either case, investment in bonds or oil is based on the expected rate of return that will prevail in the following period.

3.2. Production

Production in each region is a nested process using Constant Elasticity of Substitution (CES) production functions. This set-up allows differentiation in the degree of substitution between different intermediate goods and factors of production.

The highest level combines composite intermediate good demand with composite factor demand to produce final output. The level below this has two branches. The first aggregates various factors into the composite factor for final production. The other aggregates a composite sub-intermediate good with oil to form the composite intermediate good for final demand. Finally, the composite sub-intermediate good itself is an aggregation of agriculture, manufacturing, and services goods. Figure 4 illustrates the structure, and the equations are in Appendix I.

Of note is that the composite demand for intermediate goods depends on oil and a separate composite of the other intermediate goods. This allows the substitution between energy and other

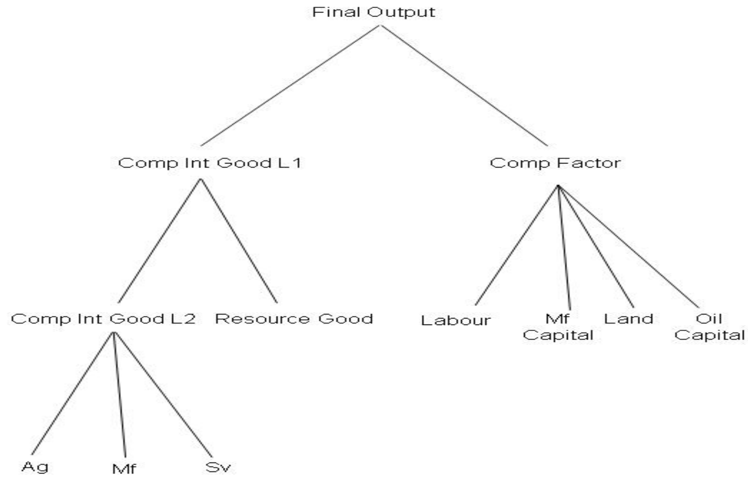

Figure 4

intermediate goods to differ from that between only the other intermediate goods. The elasticity of substitution between oil and other goods is such that any growth in this sector requires the use of additional oil, and cannot be met by substitution and additional intermediate good use.

While the oil sector is of particular importance, its structure in production is equivalent to the other industries. The differentiation will be in regards to asset portfolios and investment, where the stock of oil can be viewed as an asset.

3.3. Consumption

The representative household in each region chooses consumption of each good by maximizing a Cobb-Douglas utility function subject to their income less savings. To replicate departures from the law of one price, product differentiation by region of origin is introduced using the Armington (1969) method.

The consumer's problem is one of static maximization, to choose the consumption of each good in the current period to maximize utility. However, it is a two-step process. First, the household maximizes utility over consumption of a generic commodity, then it takes account of price differences between regions and minimizes total costs of consuming generic amounts for each good. Appendix I has the equations.

3.4. Asset Portfolios and Investment

Investment in each region comes from the savings of both regions, which is assumed to be a fixed proportion of GNP. These savings are invested in a two-step process. The first step explicitly models oil as an asset. Households in each region choose between investment in capital or oil by comparing expected rates of return. The investment in capital is financed by the issue of bonds, which define all claims over physical capital in the manufacturing sector (Pennings and Tyers, 2008). The investment in oil is modeled as a repurchase from the goods market.

The rates of return on regional bonds on the one hand, and from holding oil in the ground on the other, differ due to imperfect substitutability and preferential biases. These are assumed to reflect risk considerations not modeled explicitly. The choice takes the form of utility maximization, where each household chooses between investing in bonds or oil:

$$\max_{RI_i, BI_i} U = \left(\tau_{ri} RI_i^{-\rho_{ri}} + \tau_{bi} BI_i^{-\rho_{ri}}\right)^{\frac{-1}{\rho_{ri}}} \tag{1}$$

Subject to:

$$SV_i = s_i GNP_i = RI_i + BI_i \tag{2}$$

This is a standard CES maximization problem. In each region i, RI_i is investment in the oil, BI_i investment in bonds, τ_{ri} the weight given to the purchase of oil, τ_{bi} the weight given to the purchase of bonds, and ρ_{ri} is the CES substitution parameter on investment in either oil or bonds.

The CES weights are based on rates of return in each region, and are defined as: $\tau_{ri} = \xi_{ri} \upsilon_i^e$ and $\tau_{bi} = \xi_{bi} \delta_i^e$. ξ_{ri} is the bias in region i for oil investment, ξ_{bi} is the bias in region i for bond investment, υ_i^e is the expected rate of return on oil, and δ_i^e is the net expected rate of return on regional capital. The elasticity of substitution, σ_{ri}, is $1/(1 + \rho_{ri})$.

Solving this gives demands for oil and bond investment in each region:

$$RI_i = SV_i \left[\frac{(\xi_{ri} \upsilon_i^e)^{\sigma_{ri}}}{(\xi_{ri} \upsilon_i^e)^{\sigma_{ri}} + (\xi_{bi} \delta_i^e)^{\sigma_{ri}}}\right] \tag{3}$$

and

$$BI_i = SV_i \left[\frac{(\xi_{bi} \delta_i^e)^{\sigma_{ri}}}{(\xi_{ri} \upsilon_i^e)^{\sigma_{ri}} + (\xi_{bi} \delta_i^e)^{\sigma_{ri}}}\right] \tag{4}$$

These demands reflect differing expected rates of return and imperfect substitutability between assets. However, the expected rate of growth of the price and the expected rate of return on bonds should still be correlated.

Any savings devoted to oil investment are used in the repurchase of that asset (this is costless, so equivalent to leaving it in the ground), which takes the form (where p_{ri} is the price of oil):

$$I_{ri} = \frac{RI_i}{p_{ri}} \tag{5}$$

This amount never leaves the stock of oil in either region. A region can only reinvest in their own oil stock.

The savings devoted to bonds can purchase both home and foreign issues. These are imperfectly substitutable between the two regions and yield different rates of return on capital each period. To choose these bonds, in each period the regional household allocates total investment in bonds between those of both regions to maximize utility. This takes the form:

$$\max_{B_i, B_i^*} U = \left(\theta_i B_i^{-\rho_{bi}} + \theta_i^* B_i^{*(-\rho_{bi})}\right)^{\frac{-1}{\rho_{bi}}} \tag{6}$$

Subject to:

$$BI_i = B_i + B_i^* \tag{7}$$

where B_i is the demand in region i for the bonds issued in region i, and B_i^* is the demand in region i for the bonds issued in the other region. θ_i is the weight given in region i for the bonds of region i, and θ_i^* is the weight given in region i for the bonds of the other region.

The weights are based on expected rates of return in each region and are defined as: $\theta_i = \psi_i \delta_i^e$ and $\theta_i = \psi_i^* \delta^{*(e)}$. ψ_i is the bias in region i for the bonds of region i, ψ_i^* is the bias in region i for the bonds of the other region.

Solving this gives demands in region i for bonds of either region:

$$B_i = BI_i \left[\frac{(\psi_i \delta_i^e)^{\sigma_{bi}}}{(\psi_i \delta_i^e)^{\sigma_{bi}} + (\psi_i^* \delta_i^{*(e)})^{\sigma_{bi}}}\right] \tag{8}$$

and

$$B_i^* = BI_i \left[\frac{(\psi_i^* \delta_i^{*(e)})^{\sigma_{bi}}}{(\psi_i^* \delta_i^{*(e)})^{\sigma_{bi}} + (\psi_i \delta_i^e)^{\sigma_{bi}}}\right] \tag{9}$$

The savings committed to bond investment in each region finance purchases of capital goods that add to the capital stock. These goods are constructed from domestic output of the manufacturing and services sectors. This emphasizes the local nature of installation costs (Pennings and Tyers, 2008). Appendix I has these equations.

3.5. Dynamics

The dynamic equations in each period look one period ahead, although multiple periods are linked through these equations. Capital stock in the next period grows through investment this period, net of any depreciation.

$$k_{t+1} = k_t(1 - DEPR_t) + I_t \tag{10}$$

Additionally, with perfect foresight the expected rate of return on capital is the next period's rate of return:

$$\delta_t^e = \delta_{t+1} \tag{11}$$

If adaptive expectations are used, this becomes (where λ is the error adjustment parameter):

$$\delta_t^e = \delta_{t-1}^e + \lambda(\delta_{r,t-1} - \delta_{r,t-1}^e) \tag{12}$$

Finally, the expected rate of return on oil is:

$$\upsilon_i^e = \frac{p_{ri}^e - p_{ri}}{p_{ri}} \tag{13}$$

Equilibrium conditions are provided in Appendix I.

4. Simulations and Results

The first of two simulations highlights the role of demand growth (due to real GDP growth) in oil production and the oil price over a baseline.[5] The second simulation adds a shock to the rate of return on bonds, which allows for an estimate of the effect of the oil producer's treatment of oil as an asset. In both cases the oil price, production, and amount retained (invested) are recorded

[5]The simulations use the GEMPACK software system which is designed for solving applied general equilibrium models.

and compared to each other and a baseline. Details on the baseline database and parameters are in Appendix II and Appendix III.

Two results emerge. First, modeling the producer as arbitraging between assets impacts the price. Second, the price impact is large. In fact, the effect of declining bond yields on the real oil price is greater than the effect of increased demand. The price increase is roughly 70% when only real GDP growth is taken into consideration, and over 150% when the declining rates of return are included.

4.1. Simulations I and II

The first simulation increases real GDP in both regions by amounts similar to their averages from 1985 to 2008. This was roughly 4% per year in the non-OPEC region and 6% per year in the OPEC region (United States Department of Agriculture, 2009). The bottom line in Figure 5 shows that real oil prices rise in the OPEC region. This is because real GDP is rising, increasing demand for oil both as a final good, and as an input to production. By 2008 the magnitude is roughly 70% higher than the original value. The increase is relatively consistent each year, although it does begin to slow over time.

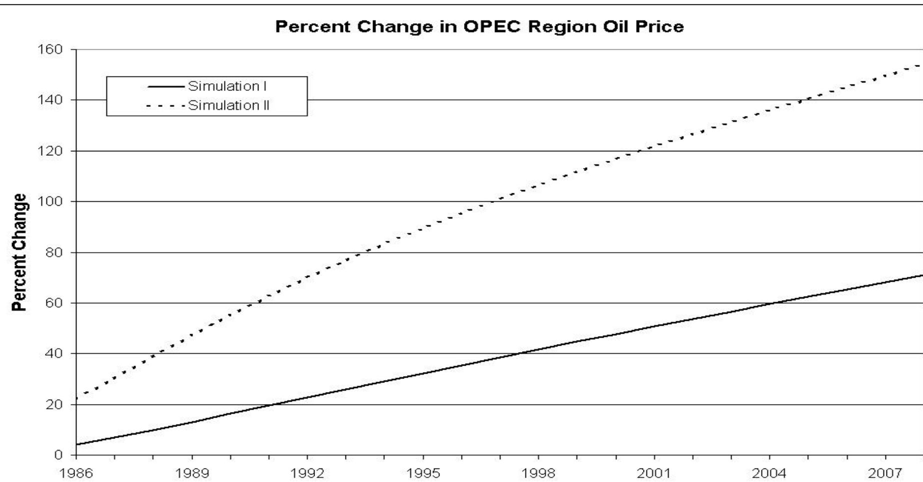

Figure 5

The top line in Figure 6 shows that oil production in this case has an initial drop and then a steady increase. This drop is attributable to the sudden price rise, whereby producers restrict production by retaining more oil in the ground. This restriction occurs because the expected rate of return on oil has risen, raising its value as an asset, inducing less production. Production begins to rise after this point due to increased demand.

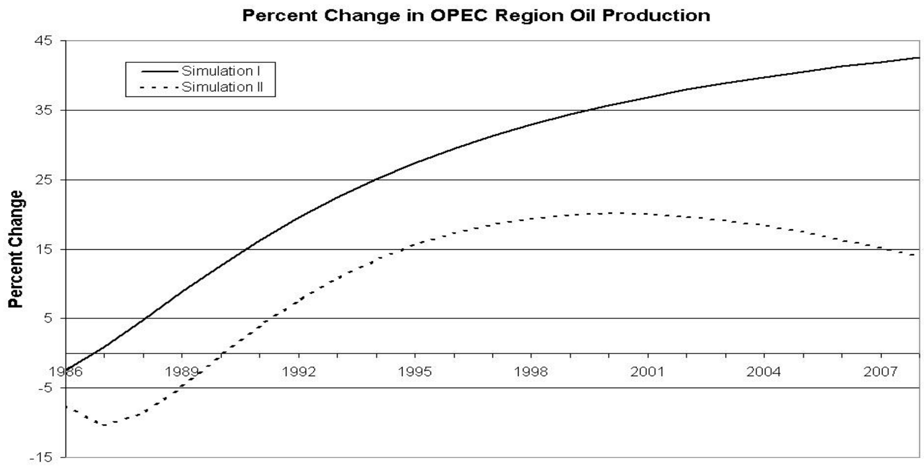

Figure 6

The lower line in Figure 7 matches this pattern. Here, investment in oil (retention of oil in the ground) rises at first and then falls. As mentioned above, the rising oil price increases expected rates of return, encouraging producers to invest more. But even though the price continues to rise, expected rates of return fall over time (because the current price is now higher), thus oil investment falls.

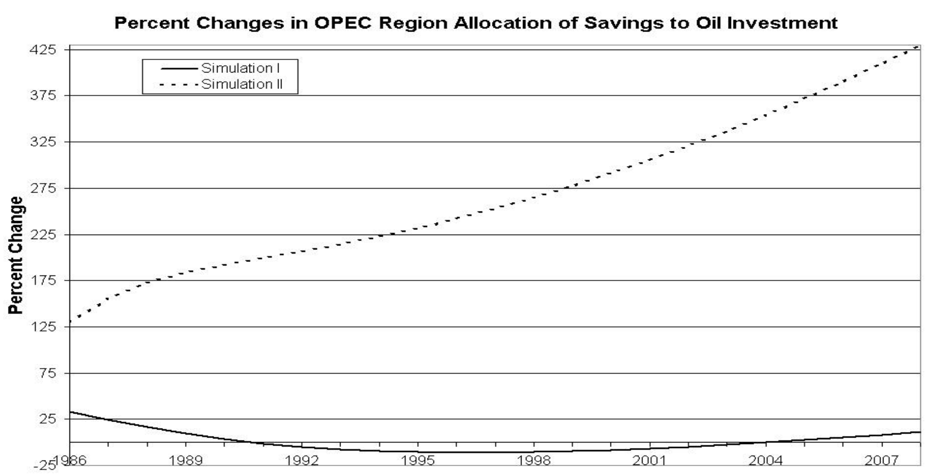

Figure 7

The second simulation matches the average historical declines in yields on 10-year U.S. Treasury Notes together with the increases in real GDP.[6] The yields decline roughly 2.5% each year on average from 1985-2008 (United States Federal Reserve, 2009). The top line in Figure 5 shows that price rises are now more pronounced. In fact, the rise in price due to falling yields is roughly 85%, while that due to demand growth is roughly 70%.

Why do falling yields have such a large impact? The key is producer arbitrage. As the rate of return on bonds falls, oil looks more favorable as an alternative investment. The top line in Figure 7 underscores this point. Even with the large price rises (which eventually contribute to a falling expected rate of return on oil), retention of oil in the ground increases. Thus the fall in yields more than offsets the fall in expected rates of return, inducing producers to retain oil. The bottom line in Figure 6 shows that production declines relative to the first simulation, driving the price higher than before.

This indicates that the desire to arbitrage asset returns may have played a significant role in recent oil price rises, possibly larger than the increases in demand. The next section discusses how sensitive the results are to various assumptions on elasticities of substitution.

4.2. Sensitivity

The model is robust to changes in the elasticity of substitution in oil investment. For the purposes of the hypothesis explored here, this is the key parameter. It encapsulates how substitutable oil and bonds are as alternative investments. The value of this parameter can drive the amount of investment into oil, thereby affecting its demand, and thus the price.

Figures 8, 9 and 10 vary the elasticity of substitution in oil investment from 1.25 to 12.5 (its value in the base simulation is 2.5) and rerun the second simulation. Figure 8 shows it makes very little difference to the oil price, which moves by less than 5% in total in each period. Figures 9 and 10 do show changes, but both eventually converge.

As the elasticity of substitution rises in Figure 9, production initially falls by more and rises by less, and then converges. At its peak, the difference between an elasticity of 1.25 with that of 12.5 is

[6] 10-year U.S. Treasury Notes are taken as an index of various investments.

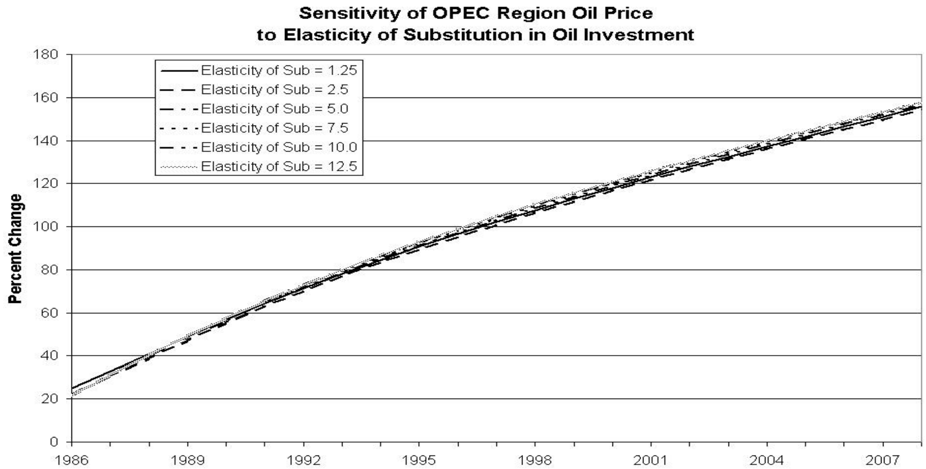

Figure 8

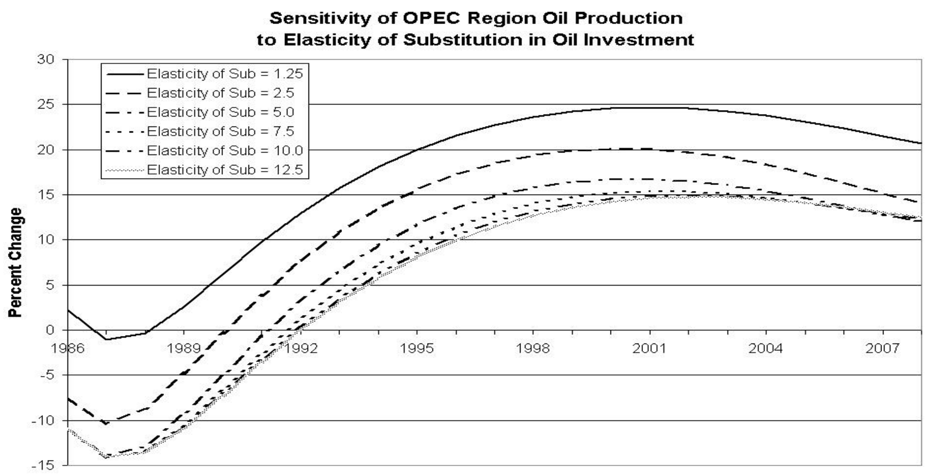

Figure 9

about 10%. As expected, oil production falls by more with a higher elasticity of substitution. This is driven by oil investment as shown in Figure 10, which rises by more as the elasticity rises. The difference in percentage terms here can be large, up to about 120% higher with a larger elasticity. Thus as bonds and oil become more substitutable as assets, there is significantly greater investment in oil because its rate of return is higher.

Importantly, the major results still hold in the face of changes in the key parameter. Clearly, other parameters are important, but none have as big an impact on the oil price, or the mechanism by which it is affected by falling rates of return.

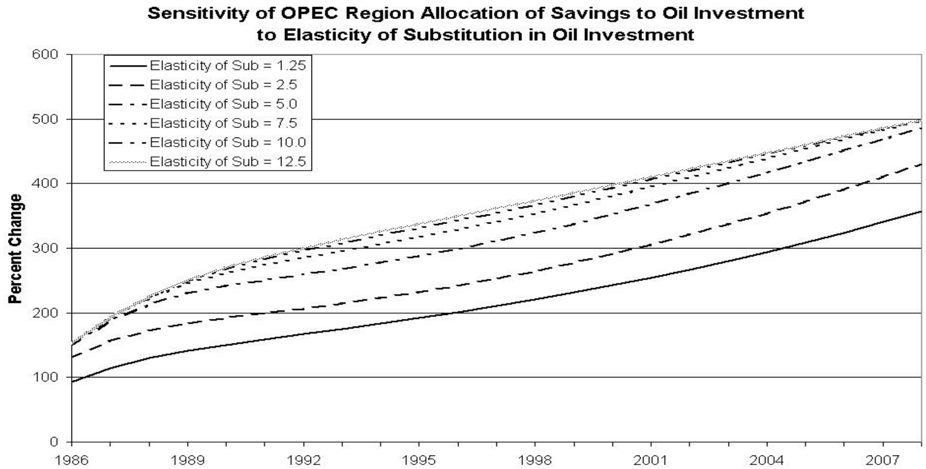

Figure 10

5. Conclusions and Discussion

There is empirical evidence of a negative relationship between the level of interest rates and the level of oil prices. This paper builds a two-region, multiple sector model which attempts to account for this behaviour. Simulations from the model show that falling interest rates raise the oil price. This is because oil in the ground is an asset to the producer, and its rate of return is compared to the interest rate. A fall in the interest rate will induce producers to reduce extraction as they seek to raise the current price. This will reduce the rate of return on oil in the ground, matching the fall in the interest rate.

How important was this relationship between interest rates and oil prices during the recent oil price boom? The simulations find that the magnitude of increases in the oil price due to supply-side responses to interest rates may have been larger than the increases in price due to rising demand. These counterfactual simulations show that increases in demand can account for roughly 70% of the price increase, whereas producer reactions to interest rates contribute approximately 85%.

The results of the paper also highlight the importance of modeling the supply-side responses of producers when oil prices are endogenous. This captures an important contributing factor in the price of oil. It also generates links between oil prices and macroeconomic variables in a more thorough way than if only the demand-side is considered.

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Appendix 1: Model Details

The model equations and specifics discussed above are provided in detail below.

Production

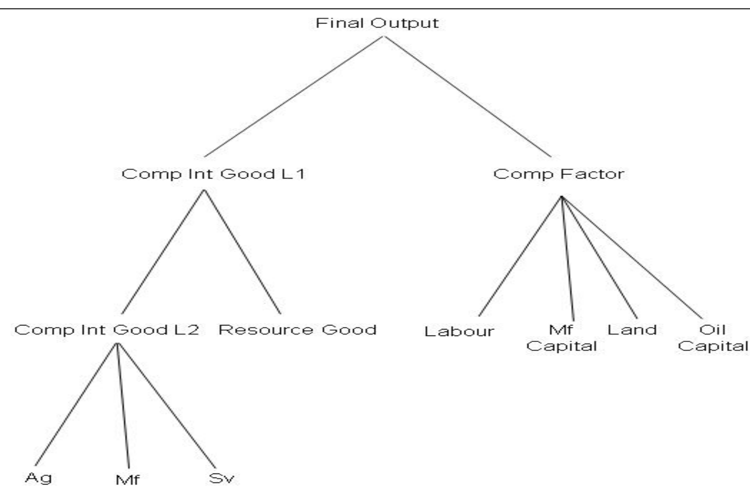

Figure 11

At the highest level, the representative firm in each region minimizes costs of production subject to a CES production function, by choosing composite quantities of factors and intermediate goods in each industry i.

$$\min_{\widehat{F_i},\widehat{Y_i}} \quad \widehat{p_{fi}}\widehat{F_i} + \widehat{p_{yi}}\widehat{Y_i} \tag{14}$$

Subject to:

$$Q_i = \widehat{A_i}\left(\varrho_{fi}\widehat{F_i}^{-\rho_{qi}} + \varrho_{yi}\widehat{Y_i}^{-\rho_{qi}}\right)^{\frac{-1}{\rho_{qi}}} \tag{15}$$

and

$$\widehat{p_{fi}} = (1/\widehat{A_{fi}})\left(\omega_{li}^{\sigma_{fi}}L^{(1-\sigma_{fi})} + \omega_{ki}^{\sigma_{fi}}K^{(1-\sigma_{fi})} + \omega_{ti}^{\sigma_{fi}}T^{(1-\sigma_{fi})} + \omega_{ji}^{\sigma_{fi}}J^{(1-\sigma_{fi})}\right)^{\frac{1}{1-\sigma_{fi}}} \tag{16}$$

and

$$\widehat{p_{yi}} = \left(1/\widehat{A_{yi}}\right)\left(\epsilon_{gi}^{\sigma_{yi}}\widehat{p_{gi}}^{(1-\sigma_{yi})} + \epsilon_{ri}^{\sigma_{yi}}\widehat{p_{ri}}^{(1-\sigma_{yi})}\right)^{\frac{1}{1-\sigma_{yi}}} \tag{17}$$

Where Q_i and $\widehat{A_i}$ are the final output and total factor productivity in final production in each sector, and $\widehat{p_{fi}}$ and $\widehat{p_{yi}}$ the composite prices of factors and intermediate goods. $\widehat{Y_i}$ and $\widehat{F_i}$ are composite intermediate good demand and composite factor demand, ω_i and ϵ_i are CES weights, and σ_i =$(1/(1+\rho_i))$ the respective elasticity of substitution. Finally, L, K, T, and J are total labor, capital, land, and oil-specific capital in each region.

The composite factor and intermediate good demands per unit of final output follow:

$$\widehat{F_i} = \left(\frac{1}{\widehat{A_i}}\right)\left[\frac{\widehat{A_i}\widehat{p_{qi}}\varrho_{fi}}{\widehat{p_{fi}}}\right]^{\sigma_{qi}} \tag{18}$$

$$\widehat{Y_i} = \left(\frac{1}{\widehat{A_i}}\right)\left[\frac{\widehat{A_i}\widehat{p_{qi}}\varrho_{yi}}{\widehat{p_{yi}}}\right]^{\sigma_{qi}} \tag{19}$$

where

$$\widehat{p_{qi}} = \left(\frac{1}{\widehat{A_i}}\right)\left(\varrho_{fi}^{\sigma_{qi}}\widehat{p_{fi}}^{(1-\sigma_{qi})} + \varrho_{yi}^{\sigma_{qi}}\widehat{p_{yi}}^{(1-\sigma_{qi})}\right)^{\frac{1}{1-\sigma_{qi}}} \tag{20}$$

Given these composite factor and intermediate good demands, each representative firm repeats the same procedure over both factors and intermediate goods in each industry i.

$$\min_{L_i,K_i,T_i,J_i} \quad w_iL_i + r_iK_i + n_iT_i + o_iJ_i \tag{21}$$

Subject to:

$$\widehat{F_i} = \widehat{A_{fi}}\left(\omega_{li}L_i^{-\rho_{fi}} + \omega_{ki}K_i^{-\rho_{fi}} + \omega_{ti}T_i^{-\rho_{fi}} + \omega_{ji}J_i^{-\rho_{fi}}\right)^{\frac{-1}{\rho_{fi}}} \tag{22}$$

This results in factor demand equations for each industry per unit factor:

$$L_i = \left(\frac{1}{\widehat{A_{fi}}}\right)\left[\frac{\widehat{A_{fi}}\widehat{p_{fi}}\omega_{li}}{w_i}\right]^{\sigma_{fi}} \tag{23}$$

$$K_i = \left(\frac{1}{\widehat{A_{fi}}}\right)\left[\frac{\widehat{A_{fi}}\widehat{p_{fi}}\omega_{ki}}{r_i}\right]^{\sigma_{fi}} \tag{24}$$

$$T_i = \left(\frac{1}{\widehat{A_{fi}}}\right)\left[\frac{\widehat{A_{fi}}\widehat{p_{fi}}\omega_{ti}}{n_i}\right]^{\sigma_{fi}} \tag{25}$$

$$J_i = \left(\frac{1}{\widehat{A_{fi}}}\right)\left[\frac{\widehat{A_{fi}}\widehat{p_{fi}}\omega_{ji}}{o_i}\right]^{\sigma_{fi}} \tag{26}$$

For intermediate goods

$$\min_{\widehat{Y_{gi}}, Y_{ri}} \quad \widehat{p_{gi}}\widehat{Y_{gi}} + \widehat{p_{ri}}Y_{ri} \tag{27}$$

Subject to:

$$\widehat{Y_i} = \widehat{A_{yi}}\left(\epsilon_{gi}\widehat{Y_{gi}}^{-\rho_{yi}} + \epsilon_{ri}Y_{ri}^{-\rho_{yi}}\right)^{\frac{-1}{\rho_{yi}}} \tag{28}$$

and

$$\widehat{p_{gi}} = (1/A_{gi})\left(\chi_{ai}^{\sigma_{gi}}\widehat{p_{ai}}^{(1-\sigma_{gi})} + \chi_{mi}^{\sigma_{gi}}\widehat{p_{mi}}^{(1-\sigma_{gi})} + \chi_{si}^{\sigma_{gi}}\widehat{p_{si}}^{(1-\sigma_{gi})}\right)^{\frac{1}{1-\sigma_{gi}}} \tag{29}$$

Where $\widehat{Y_{gi}}$ is composite demand for intermediate goods save oil, and Y_{ri} is intermediate demand for oil. This results in sub-composite intermediate good demand or oil demand per unit composite intermediate good output:

$$\widehat{Y_{gi}} = \left(\frac{1}{\widehat{A_{yi}}}\right)\left[\frac{\widehat{A_{yi}}\widehat{p_{yi}}\epsilon_{gi}}{\widehat{p_{gi}}}\right]^{\sigma_{yi}} \tag{30}$$

$$Y_{ri} = \left(\frac{1}{\widehat{A_{yi}}}\right)\left[\frac{\widehat{A_{yi}}\widehat{p_{yi}}\epsilon_{ri}}{\widehat{p_{ri}}}\right]^{\sigma_{yi}} \tag{31}$$

Where w, r, n, and o are the returns to labor, capital, land, and the oil-specific capital, and $\widehat{p_i}$ is the price of each good. Individual demands for these other intermediate goods can similarly be found:

$$\min_{Y_{ai}, Y_{mi}, Y_{si}} \quad \widehat{p_{ai}}Y_{ai} + \widehat{p_{mi}}Y_{mi} + \widehat{p_{si}}Y_{si} \tag{32}$$

Subject to:

$$\widehat{Y_{gi}} = A_{gi}\left(\chi_{ai}Y_{ai}^{-\rho_{gi}} + \chi_{mi}Y_{mi}^{-\rho_{gi}} + \chi_{si}Y_{si}^{-\rho_{gi}}\right)^{\frac{-1}{\rho_{gi}}} \tag{33}$$

The per unit intermediate good demands (save oil) are:

$$Y_{ai} = \left(\frac{1}{A_{gi}}\right)\left[\frac{A_{gi}\widehat{p_{gi}}\chi_{ai}}{\widehat{p_{ai}}}\right]^{\sigma_{gi}} \tag{34}$$

$$Y_{mi} = \left(\frac{1}{A_{gi}}\right)\left[\frac{A_{gi}\widehat{p_{gi}}\chi_{mi}}{\widehat{p_{mi}}}\right]^{\sigma_{gi}} \tag{35}$$

$$Y_{si} = \left(\frac{1}{A_{gi}}\right)\left[\frac{A_{gi}\widehat{p_{gi}}\chi_{si}}{\widehat{p_{si}}}\right]^{\sigma_{gi}} \tag{36}$$

Finally, composite prices for all intermediate goods depend on a CES combination of home and foreign varieties (where a $*$ indicates foreign).

$$\widehat{p_{xi}} = \left(\varpi_{hi}^{\sigma_{yji}} p_{xi}^{(1-\sigma_{yji})} + \varpi_{fi}^{\sigma_{yji}} p_{xi}^{\star(1-\sigma_{yji})}\right)^{\frac{1}{1-\sigma_{yji}}} \tag{37}$$

Consumption

In the first step, each regional household maximizes utility over generic commodities:

$$\max_{\widehat{C_{ai}},\widehat{C_{mi}},\widehat{C_{si}},\widehat{C_{ri}}} U(\widehat{C_{ai}},\widehat{C_{mi}},\widehat{C_{si}},\widehat{C_{ri}}) = \widehat{C_{ai}}^{\beta}\widehat{C_{mi}}^{\gamma}\widehat{C_{si}}^{\pi}\widehat{C_{ri}}^{1-\beta-\gamma-\pi} \tag{38}$$

Subject to:

$$(1-s_i)GNP_i = \widehat{p_{ai}}\widehat{C_{ai}} + \widehat{p_{mi}}\widehat{C_{mi}} + \widehat{p_{si}}\widehat{C_{si}} + \widehat{p_{ri}}\widehat{C_{ri}} \tag{39}$$

This yields consumption demand equations for each generic commodity:

$$\widehat{C}_{ai} = \left(\frac{\beta(1-s_i)GNP_i}{\widehat{p_{ai}}}\right) \tag{40}$$

$$\widehat{C}_{mi} = \left(\frac{\gamma(1-s_i)GNP_i}{\widehat{p_{mi}}}\right) \tag{41}$$

$$\widehat{C}_{si} = \left(\frac{\pi(1-s_i)GNP_i}{\widehat{p_{si}}}\right) \tag{42}$$

$$\widehat{C}_{ri} = \left(\frac{(1-\beta-\gamma-\pi)(1-s_i)GNP_i}{\widehat{p_{ri}}}\right) \tag{43}$$

s_i is the savings rate in each region, and $\hat{p}_{gi}$ is the generic price of each good in each region, which is a CES combination of home and foreign varieties:

$$\widehat{p_{gi}} = \left(\phi_{gi}^{\sigma_{ci}} p_{gi}^{(1-\sigma_{ci})} + \phi_{gi}^{\star\sigma_{ci}} p_{gi}^{\star(1-\sigma_{ci})}\right)^{\frac{1}{1-\sigma_{ci}}} \tag{44}$$

Next, the household takes account of price differences between regions and minimizes total costs of consuming generic amounts for each good:

$$\min_{C_i, C_i^*} p_i C_i + p_i^* C_i^* \tag{45}$$

Subject to:

$$\widehat{C}_i = \left(\phi_i C_i^{-\rho_{ci}} + \phi_i^* C_i^{*(-\rho_{ci})}\right)^{\frac{-1}{\rho_{ci}}} \tag{46}$$

Which yields specific demands for each good in each region:

$$C_i = (\phi_i^{\sigma_{ci}}\widehat{C}_i)\left(\frac{p_i}{\widehat{p}_i}\right)^{-\sigma_{ci}} \tag{47}$$

$$C_i^* = (\phi_i^{*(\sigma_{ci})}\widehat{C}_i^*)\left(\frac{p_i^*}{\widehat{p}_i}\right)^{-\sigma_{ci}} \tag{48}$$

C_i is consumption in region i of the good produced in the home region, C_i^* the consumption in region i of the good produced in the foreign region. p_i and p_i^* are the prices of the goods produced in the home and foreign region, ϕ_i is the weight given in region i to consumption of the good from region i, ϕ_i^* is the weight given in region i to consumption of the good from the foreign region, and ρ_{ci} is the CES substitution parameter on consumption in region i. The elasticity of substitution in consumption, σ_{ci}, is $1/(1+\rho_{ci})$.

Asset Portfolios and Investment

The composition of goods in investment is chosen by minimizing cost subject to a CES installation function in each region:

$$\min_{I_{mi},I_{si}} p_{mi}I_{mi} + p_{si}I_{si} \tag{49}$$

Subject to:

$$\left(\eta_{mi}I_{mi}^{-\rho_{Ii}} + \eta_{si}I_{si}^{-\rho_{Ii}}\right)^{\frac{-1}{\rho_{Ii}}} \tag{50}$$

η_{si} is the weight of investment given to services, η_{mi} is the weight of investment given to manufacturing goods, and ρ_{Ii} the CES substitution parameter on local investment in region i. The optimization produces demands for manufacturing and services goods in producing capital. It also gives the price of capital goods, which are a composite of the price of manufactured goods and services in each region.

The investment demands are:

$$I_{mi} = (\eta_{mi}^{\sigma_{Ii}}I_i)\left(\frac{p_{mi}}{\widehat{p_{ki}}}\right)^{-\sigma_{Ii}} \tag{51}$$

$$I_{si} = (\eta_{si}^{\sigma_{Ii}}I_i)\left(\frac{p_{si}}{\widehat{p_{ki}}}\right)^{-\sigma_{Ii}} \tag{52}$$

where

$$I_i = \frac{B_i + B_i^*}{\widehat{p_{ki}}} \tag{53}$$

and the price of capital:

$$\widehat{p_{ki}} = \left(\eta_{mi}^{\sigma_{Ii}}p_{mi}^{(1-\sigma_{Ii})} + \eta_{si}^{\sigma_{Ii}}p_{si}^{(1-\sigma_{Ii})}\right)^{\frac{1}{1-\sigma_{Ii}}} \tag{54}$$

Rates of Return

The rate of return on capital in either region is the real rate of return less depreciation in the current period:

$$\delta_i = \frac{r_i}{\widehat{p_{ki}}} - DEPR_i \tag{55}$$

The oil rate of return is the rate of growth in the actual oil price. The actual price is used instead of the shadow price (or value in the ground) because it is assumed there is no scarcity rent associated with the oil. This is an expected rate of return because the future price is not known today.

$$\upsilon_i^e = \frac{p_{ri}^e - p_{ri}}{p_{ri}} \tag{56}$$

Only the home price of the oil enters this expected return.

Equilibrium Conditions

The model is closed with a series of equilibrium conditions in each market. First, for each good output must equal domestic consumption, domestic intermediate use, domestic investment use, and exports (note that one condition is dropped in one region due to Walras' Law):

$$Q_{ai} = C_{ai} + EX_{ai} + Y_{ai} \tag{57}$$

$$Q_{mi} = C_{mi} + EX_{mi} + Y_{mi} + I_{mi} \tag{58}$$

$$Q_{si} = C_{si} + EX_{si} + Y_{si} + I_{si} \tag{59}$$

$$Q_{ri} = C_{ri} + EX_{ri} + Y_{ri} + I_{ri} \tag{60}$$

The factor market equilibrium conditions say that each factor is limited to its use in any domestic industry:

$$L_i = L_{ai} + L_{mi} + L_{si} + L_{ri} \tag{61}$$

$$K_i = K_{mi} \tag{62}$$

$$T_i = T_{ai} \tag{63}$$

$$J_i = J_{ri} \tag{64}$$

Finally, the quantity of trade must balance (not necessarily the value, or current account). So the exports of one country equal the imports of the other, and vice versa:

$$EX_i = IM_j \tag{65}$$

Appendix 2: Data and Calibration

The data used in simulations are from the Global Trade Analysis Project (2007). They are representative of Middle East OPEC countries and the rest of the world, and represent a reference year of 2001, with money values in 2001 U.S. dollars. Specifically, Region 1 includes data on the following OPEC members: Saudi Arabia, Iran, Iraq, The United Arab Emirates, Kuwait, and Qatar. Non-OPEC members in Region 1 include Bahrain, Israel, Jordan, Lebanon, Palestine, Oman, Syria, and Yemen. The non-OPEC members are included due to the particularities of GTAP regional classifications.

In constructing the initial input/output table, the values of certain quantities at market prices were taken from the GTAP database for each region. These were then put into percentage values in order to use proportional changes instead of actual levels values. For example, Region 2 GDP is roughly 45 times the GDP of Region 1. Region 1 GDP was then set arbitrarily at 1000, and Region 2 at 45000. Given this baseline, each of the data series categories were allocated as a percent of this GDP, proportional to its total value.

The specific data series from the GTAP database were: VALOUTOUT (the value of output), OUTDISP (disposition of output), DOMSALESDISP (the disposition of domestic sales), IMPSALESDISP (the disposition of imported goods), VDFM (domestic intermediate good demand), VIFM (import intermediate good demand), VDPM (domestic consumption), VIPM (import consumption), VALEXPORTS (the value of exports), VALIMPORTS (the value of imports), and VFM (the value of endowments). Additionally, a savings rate was inferred from the GTAP series SAVE (net savings), and GDPEXP (GDP by expenditures). This gave a value of savings in Region 1 slightly greater than Region 2. However, to better illustrate differences between OPEC and the ROW, the savings rate in Region 1 was set at roughly 2.5 times that of Region 2 (roughly OPEC Middle East savings over the ROW on average).

The input/output table was then balanced using the standard RAS method. This yielded the values that constitute an equilibrium in the base year. Certain parameters, such as CES weights, were then calibrated from this equilibrium data. Other parameter values, such as elasticities of substitution, were based on similarities with other models, or expected values based on intuition. The key elasticities between oil and other intermediate goods in production are very low in both countries, at 0.05. The elasticities are summarized below in Tables II and III.

The modeling software used is GEMPACK, which requires a full solution for each period under consideration. The dynamic nature of the model makes it a relatively complex process to build a multi-period database that is consistent with the original, given that future data are unknown. This is done by solving the model all-at-once (instead of recursively) in the GEMPACK software suite of WINGEM using the method of Wendner (1999). The static base case is taken and extrapolated over the entire model horizon. This is done by adding a slack term to each dynamic equation, and then running a simulation that shocks each slack term down to zero. This builds a database that is consistent with the dynamic equations of the model. However, it may not exactly replicate the original equilibrium.

The last step is to endogenize certain select parameters (such as the discount factor or depreciation), and to shock the values that are not the same as the original equilibrium (in the initial period) to their respective values. The result is a multi-period database which is consistent with both the dynamic equations of the model and the original equilibrium data. This method was used to build a steady state database in the base year, which allows all shocks to be easily compared with the baseline scenario.

Once the database was built, the specified shocks were applied. The shocks were designed to decompose salient features of oil prices and depended on historical data on real GDP and interest rates. The real historical real GDP data was taken from United States Department of Agriculture (2009) and the interest rate data from United States Federal Reserve (2009). In both cases, 1985 is taken as a base year and the shocks applied are deviations from this value. The shocks themselves are average changes in growth or declines of the variable. For example, the shocks to real GDP average growth of nearly 4% for non-OPEC countries from 1985-2008. The same method was applied to interest rates, where the average declines in Region 2 were 2.5%. While not exact, the magnitude and trend of the shocks is enough to simulate behavior given that the base data itself is an approximation.

Appendix 3: Parameters

Table I: Elasticities of Substitution

Parameter	Description
σ_{qi}	Region i elasticity of sub in prod between int goods/factors
σ_{fi}	Region i elasticity of substitution in production between factors
σ_{yi}	Region i elasticity of sub in prod between comp int goods and oil
σ_{igi}	Region i elasticity of sub in production between int goods
σ_{yji}	Region i elasticity of sub between H/F for each int good or oil
σ_{ci}	Region i elasticity of sub in cons between types of goods from H/F
σ_{ri}	Region i elasticity of sub in investment between oil and bonds
σ_{bi}	Region i elasticity of sub in investment between bonds issued by H/F
σ_{Ii}	Region i elasticity of sub in capital construction

Table II: OPEC Region Parameter Values

Parameter	I1	I2	I3	I4
σ_{q1}	0.75	0.75	0.75	0.05
σ_{f1}	1.0	1.0	1.0	1.0
σ_{y1}	0.05	0.05	0.05	0.05
σ_{ig1}	1.0	1.0	1.0	–
σ_{c1}	5.0	5.0	0.5	100
σ_{yj1}	5.0	5.0	0.5	100
Regional Values				
σ_{r1}	2.5			
σ_{b1}	2.5			
σ_{I1}	2.5			
Saving rate	0.19			
Depreciation rate	0.05			

Table III: non-OPEC Region Parameter Values

Parameter	I1	I2	I3	I4
σ_{q2}	0.75	0.75	0.75	0.05
σ_{f2}	1.0	1.0	1.0	1.0
σ_{y2}	0.05	0.05	0.05	0.05
σ_{ig2}	1.0	1.0	1.0	–
σ_{c2}	5.0	5.0	0.5	100
σ_{yj2}	5.0	5.0	0.5	100
Regional Values				
σ_{r2}		2.5		
σ_{b2}		2.5		
σ_{I2}		2.5		
Saving rate		0.08		
Depreciation rate		0.05		